# Newton's Method

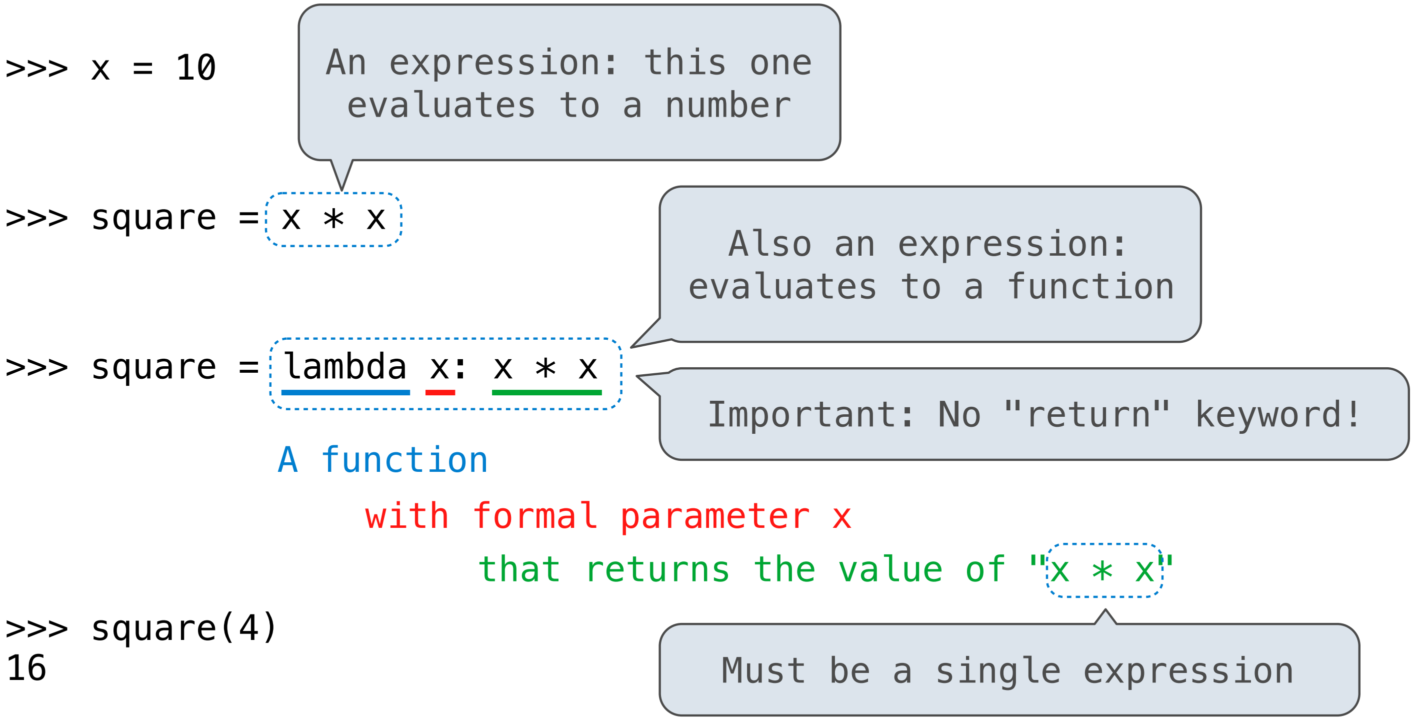
# Announcements

- If you want 1 unit (pass/no pass) of credit for in CS 98-52, the CCN is 28867
  - Only for people who really want extra work that's beyond the scope of normal CS 61A
- Anyone is welcome to attend the extra lectures, whether or not they enroll
- All info and materials will be posted to cs61a.org/extra.html

# Lambda Expressions

(Demo)

# Lambda Expressions



```
>>> x = 10

>>> square = x * x

>>> square = lambda x: x * x

>>> square(4)
16
```

An expression: this one evaluates to a number

Also an expression: evaluates to a function

Important: No "return" keyword!

A function

with formal parameter x

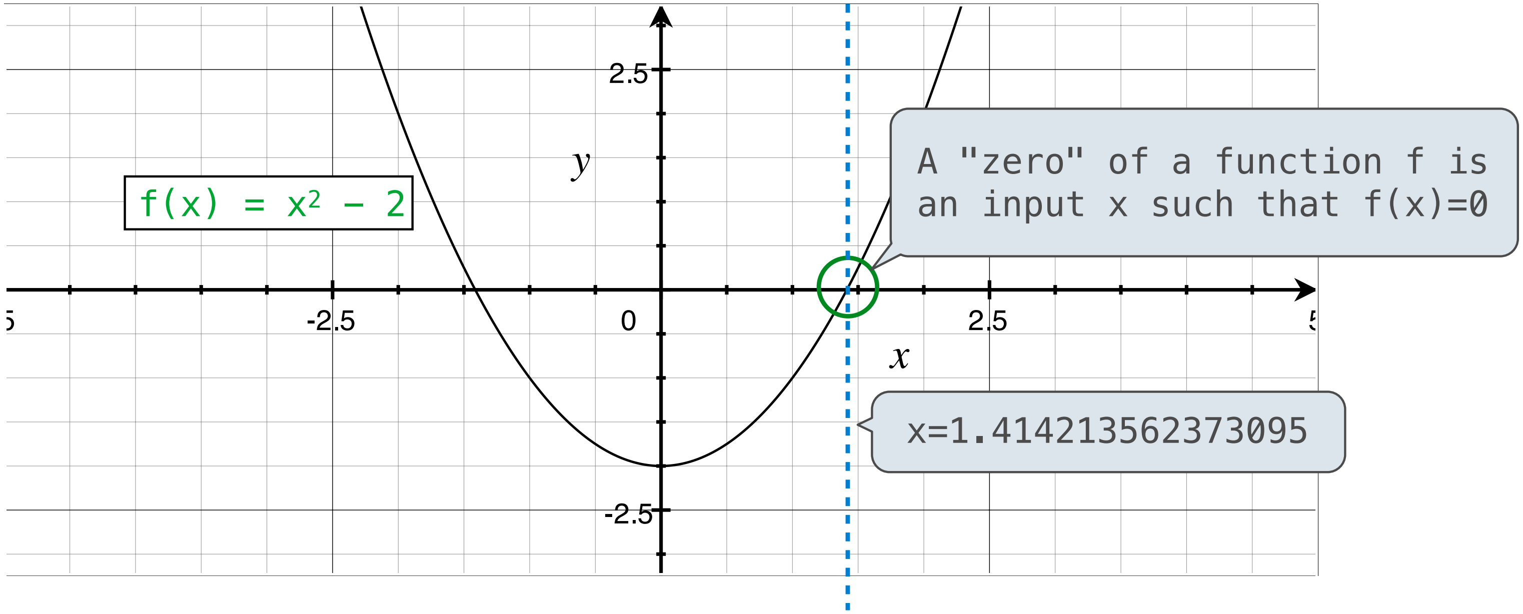
that returns the value of "x * x"

Must be a single expression

# Newton's Method

# Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!



A "zero" of a function f is an input x such that f(x)=0

x=1.414213562373095

Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)

# Newton's Method

Given a function f and initial guess x,

Repeatedly improve x:

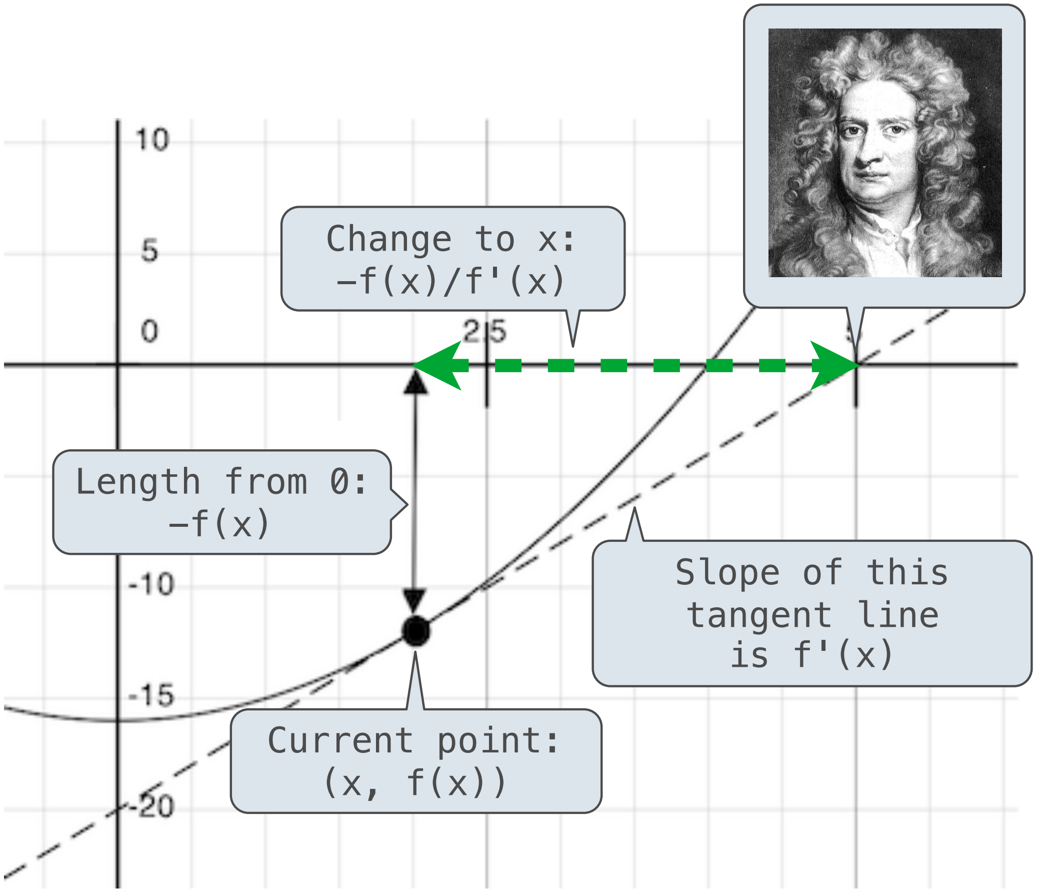
- Compute the value of f at the guess: f(x)

- Compute the derivative of f at the guess: f'(x)

- Update guess x to be:

$$x - \frac{f(x)}{f'(x)}$$

Finish when f(x) = 0 (or close enough)



Change to x: −f(x)/f'(x)

Length from 0: −f(x)

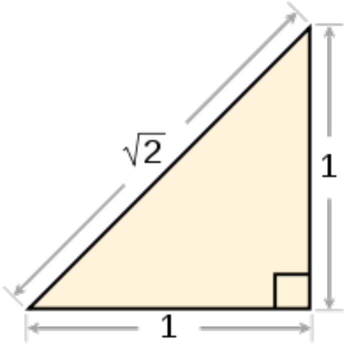
Slope of this tangent line is f'(x)

Current point: (x, f(x))

http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

# Using Newton's Method

How to find the square root of 2?

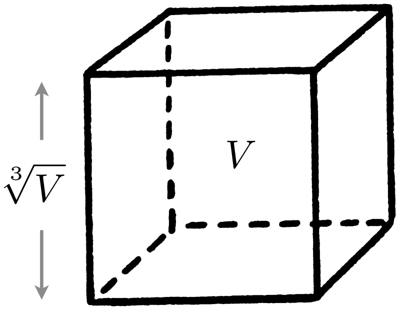


```
>>> f  = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
```

$f(x) = x^2 - 2$
$f'(x) = 2x$

Applies Newton's method

How to find the cube root of 729?



```
>>> g  = lambda x: x*x*x - 729
>>> dg = lambda x: 3*x*x
>>> find_zero(g, dg)
9.0
```

$g(x) = x^3 - 729$
$g'(x) = 3x^2$

# Iterative Improvement

## Special Case: Square Roots

How to compute square_root(a)

**Idea:** Iteratively refine a guess x about the square root of a

**Update:** $$x = \frac{x + \frac{a}{x}}{2}$$ (Demo)

Babylonian Method

**Implementation questions:**

What guess should start the computation?

How do we know when we are finished?

# Special Case: Cube Roots

How to compute cube_root(a)

**Idea:** Iteratively refine a guess x about the cube root of a

**Update:**

$$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$

(Demo)

**Implementation questions:**

What guess should start the computation?

How do we know when we are finished?

# Implementing Newton's Method

(Demo)

# Extensions

# Approximate Differentiation

Differentiation can be performed symbolically or numerically

f(x) = $x^2$ − 16

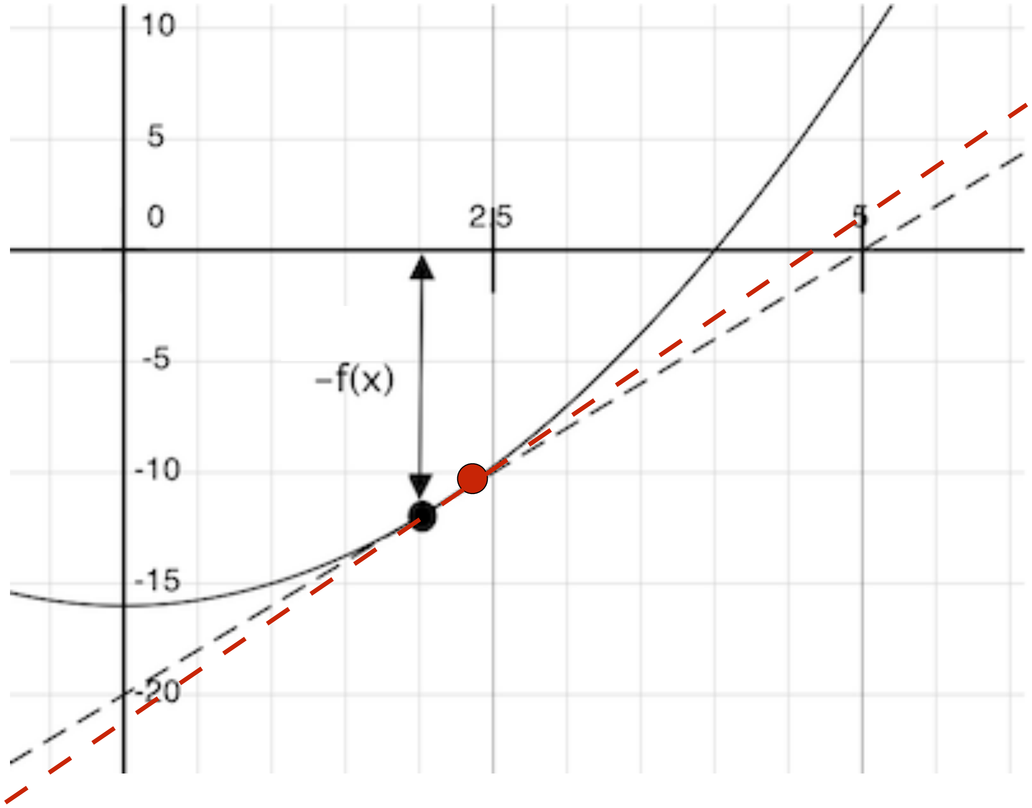
f'(x) = 2x

f'(2) = 4

$$f'(x) = \lim_{a \to 0} \frac{f(x+a) - f(x)}{a}$$

$$f'(x) \approx \frac{f(x+a) - f(x)}{a}$$ (if $a$ is small)

(Demo)



# Inverse Function

The inverse $f^{-1}(y)$ of a differentiable, one-to-one function computes the value x such that $f(x) = y$

(Demo)