

INSTRUCTIONS

- You have 3 hours to complete the exam.
- The exam is closed book, closed notes, closed computer, closed calculator, except three hand-written 8.5" × 11" crib sheet of your own creation and the official CS 61A midterm 1, midterm 2, and final study guides.
- Mark your answers **on the exam itself**. We will *not* grade answers written on scratch paper.

Last name	
First name	
Student ID number	
CalCentral email (_@berkeley.edu)	
TA	
Name of the person to your left	
Name of the person to your right	
<i>All the work on this exam is my own.</i> (please sign)	

POLICIES & CLARIFICATIONS

- If you need to use the restroom, bring your phone and exam to the front of the room.
- You may use built-in Python functions that do not require import, such as `min`, `max`, `pow`, `len`, `abs`, `sum`, `next`, `iter`, `list`, `tuple`, `map`, `filter`, `zip`, `all`, and `any`.
- You **may not** use example functions defined on your study guides unless a problem clearly states you can.
- For fill-in-the-blank coding problems, we will only grade work written in the provided blanks. You may only write one Python statement per blank line, and it must be indented to the level that the blank is indented.
- Unless otherwise specified, you are allowed to reference functions defined in previous parts of the same question.
- You may use the `Tree`, `Link`, and `BTree` classes defined on Page 2 (left column) of the Midterm 2 Study Guide.

1. (12 points) High Quality Air

For each of the expressions in the table below, write the output displayed by the interactive Python interpreter when the expression is evaluated. The output may have multiple lines. The first row is completed for you.

- If an error occurs, write **ERROR**, but include all output displayed before the error.
- To display a function value, write **FUNCTION**.
- To display an iterator value, write **ITERATOR**.
- If an expression would take forever to evaluate, write **FOREVER**.

The interactive interpreter displays the contents of the `repr` string of the value of a successfully evaluated expression, unless it is `None`.

Assume that you have started `python3` and executed the code shown on the left first, then you evaluate each expression on the right in the order shown. Expressions evaluated by the interpreter have a cumulative effect.

```
from operator import sub

z = (lambda x: lambda y: 2 * (y-x))(3)

def breath(f, count=1):
    if count > 1:
        print(count)
    count += 1
    return lambda x, y: f(x+1, y)

class Day:
    aqi = 10
    def __init__(self, aqi=0):
        if aqi > self.aqi:
            self.aqi = aqi
        self.n = []
    def mask(self, limit):
        def f(aqi):
            if aqi > limit:
                self.n.append(aqi-limit)
            return self.mask(aqi)
        return f

class Week(Day):
    aqi = 50

m, t = Day(), Week(199)
t.mask(200)(100)(150)(160)
Day.aqi = 140
t.aqi = 160
```

Expression	Output
<code>print(None)</code>	None
<code>print(print(None), print)</code>	None None Function
<code>z(4)</code>	2
<code>breath(breath(sub))(5, 3)</code>	4
<code>[Day().aqi, m.aqi]</code>	[140, 140]
<code>[Week.aqi, t.aqi]</code>	[50, 160]
<code>t.n</code>	[50, 10]

3. (16 points) Gainz

Definition. A sequence is *near increasing* if each element beyond the second is larger than all elements preceding its previous element. That is, element i must be larger than elements $i - 2$, $i - 3$, $i - 4$, etc.

- (a) (3 pt) Implement `is_near`, which takes a sequence `s` and returns whether its elements form a near increasing sequence.

```
def is_near(s):
    """Return whether s is a near increasing sequence.

    >>> is_near([]) and is_near([1]) and is_near([1, 2]) and is_near(range(10))
    True
    >>> is_near([4, 2]) and is_near((1, 4, 2, 5)) and is_near((1, 2, 4, 3))
    True
    >>> is_near((3, 2, 1))          # 1 <= 3
    False
    >>> is_near([1, 4, 2, 3, 5])    # 3 <= 4
    False
    >>> is_near([1, 4, 2, 5, 3])    # 3 <= 4
    False
    >>> is_near([1, 2, 4, 2, 5])    # 2 <= 2
    False
    """

    return all([s[i] > max(s[:i-1]) for i in range(2, len(s))])
```

- (b) (6 pt) Implement `fast_near`, which takes an iterable value and returns whether its elements form a near increasing sequence. `fast_near` must run in $\Theta(n)$ time and $\Theta(1)$ space (not including the input itself) for an iterable input with n elements. Assume that `s` has a finite number of elements. You may not call `is_near`.

```
def fast_near(s):
    """Return whether s is a near increasing sequence.

    >>> fast_near([2, 5, 3, 6, 6, 7, 7, 9, 8])
    True
    """
    t, s = iter(s), None # Do not refer to s below this line.

    try:

        largest, last = next(t), next(t)

    except StopIteration:

        return True

    for x in t:

        if x <= largest:

            return False

        largest, last = max(largest, last), x

    return True
```

Alternative Definition. (Equivalent to the one on the previous page, but stated in a more useful way for the problem below.) A sequence is *near increasing* if each element but the last two is smaller than all elements following its subsequent element. That is, element i must be smaller than elements $i + 2$, $i + 3$, $i + 4$, etc.

- (c) (6 pt) Implement `near`, which takes a non-negative integer `n` and returns the largest near increasing sequence of digits within `n` as an integer. The arguments `smallest` and `d` are part of the implementation; you must determine their purpose. You may not call `is_near` or `fast_near`. You may not use any values except integers and booleans (`True` and `False`) in your solution (no lists, strings, etc.).

```
def near(n, smallest=10, d=10):
    """Return the longest sequence of near-increasing digits in n.

    >>> near(123)
    123
    >>> near(153)
    153
    >>> near(1523)
    153
    >>> near(15123)
    1123
    >>> near(11111111)
    11
    >>> near(985357)
    557
    >>> near(14735476)
    143576
    >>> near(812348567)
    1234567
    """

    if n == 0:

        return 0

    no = near(n//10, smallest, d)

    if smallest > n % 10:

        yes = 10 * near(n//10, d, min(d, n%10)) + n%10

        # OR yes = 10 * near(n//10, min(smallest, d), n%10) + n%10

        return max(yes, no)

    return no
```

- (d) (1 pt) What is the largest possible integer that could ever be returned from the `near` function? **Note:** In general, integers in Python can be arbitrarily large.

112233445566778899

4. (11 points) Tree Time

Definition. A *runt node* is a node in a tree whose label is smaller than all of the labels of its siblings. A sibling is another node that shares the same parent. A node with no siblings is a runt node.

- (a) (7 pt) Implement `runts`, which takes a `Tree` instance `t` in which *every label is different* and returns a list of the labels of all runt nodes in `t`, in any order. Also implement `apply_to_nodes`, which returns nothing and is part of the implementation. Do **not** mutate any tree. The `Tree` class is on the Midterm 2 Guide.

```
def runts(t):
    """Return a list of the labels of all smallest siblings in any order.

    >>> sorted(runts(Tree(9, [Tree(3), Tree(4, [Tree(5, [Tree(6)])], Tree(7)]), Tree(2))))
    [2, 5, 6, 9]
    """
    result = []

    def g(node):
        if not node.is_leaf():
            result.append(min([b.label for b in node.branches]))

    apply_to_nodes(g, t)

    return [t.label] + result

def apply_to_nodes(f, t):
    """Apply a function f to each node in a Tree instance t."""

    f(t)

    for b in t.branches:

        apply_to_nodes(f, b)
```

- (b) (4 pt) Implement `max_label`, which takes a `Tree` `t` and returns its largest label. Do **not** mutate any tree.

```
def max_label(t):
    """Return the largest label in t.

    >>> max_label(Tree(4, [Tree(5), Tree(3, [Tree(6, [Tree(1), Tree(2)])])]))
    6
    """
    def f(node):
        nonlocal t

        t = max(t, node, key=lambda n: n.label)

    apply_to_nodes(f, t)

    return t.label
```

5. (9 points) Run, Program, Run

Implement `runs`, a Scheme procedure that takes a list of integers `s` and returns a list of non-empty lists of integers `t`. Together, the lists in `t` should contain all elements of `s` in order. The first element in each list in `t` must be less than the last element in the previous list, if there is one. The rest of the elements in each list in `t` must be greater than or equal to the previous element.

Also implement and use `next-run` in your solution, which takes a non-empty list of integers `s` and returns a pair of lists: the longest non-decreasing prefix of `s` and the rest of `s`. Use the provided `pair` data abstraction. Your implementation should be correct even if the `pair` implementation were to change.

```
;; Return a list of non-decreasing lists that together contain the elements of s.
;; scm> (runs '(3 4 7 6 6 8 1 2 5 5 4))
;; ((3 4 7) (6 6 8) (1 2 5 5) (4))
;; scm> (runs '(4 3 2 3))
;; ((4) (3) (2 3))
(define (runs s)
```

```
  (if (null? s) nil
```

```
      (let ((p (next-run s)))
```

```
          (cons (first p) (runs (rest p))))))
```

```
;; A data abstraction for a pair of a first run and the rest of a list.
```

```
(define (pair a b) (lambda (c) (if c a b)))
```

```
(define (first p) (p #t))
```

```
(define (rest p) (p #f))
```

```
;; Return a pair containing the first run in s (a list) and the rest of s (another list).
```

```
;; scm> (first (next-run '(4 5 1 3 2)))
```

```
;; (4 5)
```

```
;; scm> (rest (next-run '(4 5 1 3 2)))
```

```
;; (1 3 2)
```

```
(define (next-run s)
```

```
  (if (or (null? (cdr s))
```

```
      (< (car (cdr s)) (car s)))
```

```
      (pair (list (car s)) (cdr s))
```

```
      (begin
```

```
        (define p (next-run (cdr s)))
```

```
        (pair (cons (car s) (first p)) (rest p))))))
```

6. (9 points) Generation Z

- (a) (4 pt) Implement `rev`, a generator function that takes a `Link` instance and yields the elements of that linked list in reverse order. The `Link` class appears on Page 2 of the Midterm 2 Study Guide.

```
def rev(s):
    """Yield the elements in Link instance s in reverse order.

    >>> list(rev(Link(1, Link(2, Link(3)))))
    [3, 2, 1]
    >>> next(rev(Link(2, Link(3))))
    3
    """
    if s is not Link.empty:

        yield from rev(s.rest)

        yield s.first
```

- (b) (2 pt) Using the provided `add` procedure, define `not-three`, an infinite stream of all positive integers that are not evenly divisible by 3. The `not-three` stream is increasing and begins with 1, 2, 4, 5, 7, 8, 10, 11, 13.

```
(define (add k s) (cons-stream (+ k (car s)) (add k (cdr-stream s))))

(define not-three (cons-stream 1 (cons-stream 2 (add 3 not-three))))
```

- (c) (3 pt) Implement `infix`, a Scheme macro that evaluates infix expressions. An infix expression is either a number or a three-element list containing an infix expression, a procedure, and another infix expression. The value of a compound infix expression is the value of its second element applied to the values of its first and third elements. **Note:** The last line begins with a quasiquote. If you cross out the quasiquote and solve the problem without using quasiquote or unquote, you can receive up to 2 out of 3 points (not recommended).

```
;; A macro to evaluate infix expressions.
;; scm> (infix (2 * 3))
;; 6
;; scm> (infix ((1 + 1) * (1 + 2)))
;; 6
;; scm> (infix ((1 + (3 - 2)) * ((2 + 3) + 2)))
;; 14
(define-macro (infix e)

  (if (number? e) e

      `(,(cadr e) (infix ,(car e)) (infix ,(caddr e)))))

(define (cadr x) (car (cdr x)))
(define (caddr x) (car (cdr (cdr x))))
```

7. (10 points) SQL of Course

The **courses** table describes the **course** name, start time hour (**h**) and minute (**m**), and length in minutes (**len**) for different lectures. For example, 61A starts at 13:00 and lasts 50 minutes. The **locations** table describes the **course name** and location (**loc**) of these courses. Assume that each course name appears exactly once in each table. Write your SQL statements so that they would still be correct if the table contents changed.

```
CREATE TABLE courses AS
```

```
SELECT "1" AS course, 14 AS h, 0 AS m, 80 AS len UNION
SELECT "2"           , 13     , 30     , 80     UNION
SELECT "8"           , 12     , 30     , 50     UNION
SELECT "10"          , 12     , 30     , 110    UNION
SELECT "50AC"        , 13     , 30     , 45     UNION
SELECT "61A"         , 13     , 0      , 50;
```

```
CREATE TABLE locations AS
```

```
SELECT "1" AS name, "VLSB" AS loc UNION
SELECT "2"           , "Dwinelle"  UNION
SELECT "10"          , "VLSB"       UNION
SELECT "50AC"        , "Wheeler"    UNION
SELECT "61A"         , "Wheeler";
```

- (a) (2 pt) Select a one-column table that contains the course names of all courses that start **before** 13:30.

```
SELECT course FROM courses WHERE h < 13 OR (h=13 AND m<30);
```

61A
8
10

- (b) (4 pt) Select a two-column table with one row per location that contains the location, as well as the shortest length in minutes of any lecture held in that location.

```
SELECT loc, min(len) FROM courses, locations WHERE name=course GROUP BY loc;
```

Dwinelle	80
VLSB	80
Wheeler	45

- (c) (4 pt) Select a three-column table where each row describes an earlier course, a later course, and the amount of time in minutes between the end time of the earlier course and the start time of the later course. Only include pairs of courses where the lectures do not overlap in time. *Note:* There are 60 minutes in an hour.

```
SELECT a.course, b.course,
```

```
(b.h - a.h) * 60 + b.m - a.m - a.len AS gap
```

```
FROM courses as a, courses as b WHERE gap > 0
```

61A	1	10
8	1	40
8	2	10
8	50AC	10

8. **(0 points)** **Draw!** (*Optional*) Draw a picture of some function or procedure.